ASYMPTOTIC ANALYSIS OF GAS COMPRESSION PROCESSES WITH SPHERICAL SYMMETRY

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In studying problems of thermonuclear synthesis with an inertial content of material, use of the following model [1-5] has proven effective: A spherical mass of material is compressed by the action of an external piston, with the quality of the compression being described by the integral parameter $\langle \rho R\rangle = \int\limits_0^r \rho dr$. The law for change of pressure on the piston has been found to agree well with the law of change of intensity of the factor acting (for example, laser radiation), and a larger value of the parameter $\langle \rho R\rangle$ corresponds to better conditions for the synthesis reaction. To describe the process mathematically the equations of an ideal compressible fluid are used together with the equation of state of an ideal gas.

A problem which develops when this approach is used, interesting from the hydrodynamic viewpoint, is the following: to find a law of piston motion, such that for given values of applied energy and compressible material mass the product $\langle \rho R\rangle$ is maximized. Solution of this problem depends significantly on the possibility of creating initial distributions of hydrodynamics functions over target mass, necessary for existence of the solution. It was shown in [5] that the parameter $\langle \rho R\rangle$ may reach infinite values for finite values of energy and compressible material mass, but at $\gamma > \gamma_*$, where $\gamma_* = 1.31$ for a collapsing shell, and $\gamma_* = 1.34$ for a solid target, this requires a special distribution of entropy over target radius. At values of $\gamma < \gamma_*$ solutions with infinite increase in $\langle \rho R\rangle$ are also possible for entropy constant over the entire mass. Since the value $\gamma = {}^5/_3 > \gamma_*$ with entropy constant over radius is of greatest interest, together with the simplest possible initial conditions, i.e., constant values of density ρ and pressure p with zero velocity u, it is necessary to determine how $\langle \rho R\rangle$ may be maximized in this case. A rigorous analytical solution of this problem is hardly possible, so that results obtained with certain simplifying assumptions take on interest.

We will assume that the distribution of gasdynamic functions over target radius in the final stage of compression (t = 0) may be approximated by power functions, in particular, $\rho = c/r^\alpha$. Then in the vicinity of the origin (r = 0, t = 0) the solution is self-similar [5, 6] and a set of α values can be found for which the final state under consideration may be derived continuously from some initial state, i.e., the compression process is physically realizable. Thus, it will be possible to obtain an asymptote at the center for all possible modes of compression with constant entropy. The system of ordinary differential equations describing self-similar solutions, dependent on the single variable $\lambda = r/t^\delta$, was presented in [6]; we will only note that isentropic flow corresponds to a value of the constant $\varkappa$ in this system which satisfies the expression $\varkappa = 2(1-\delta)/(\gamma-1)$, with the relationship between α and δ being $\alpha = \varkappa/\delta$. A study of singular points in the plane of the self-similar variables (z, V) reveals that there exists a characteristic value α_* [5, 7]:

$$\alpha_* = 2(3-\sqrt{3})/(\sqrt{3}(\gamma-1)+2),$$

which is the maximum possible at $\gamma = {}^5/_3$. A final state for the entire range of exponent values $\alpha < \alpha_*$ for this γ value can be reached by different methods: for the collapsing shell regime [7] with a nonanalytic, generally speaking, transition through a singular characteristic; for compression of a solid target, which corresponds to a self-similar curve going from the origin to the singular saddle point $B[V = 2/(3\gamma-1),\ z = 3(\gamma-1)^2/(3\gamma-1)^2]$. Solid target compression also corresponds to a set of integral curves in a sheath of curves located above the separatrix referred to above and passing through a node type singular point on the parabola $z = (V-\delta)^2$, from which the separatrix of the saddle point ($V = \varkappa/3$, $z = \infty$),

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exits, describing compression of the central portion of the target. At $\gamma = {}^5/_3$, as was shown in [7], a solution with an internal cavity is possibly only for $\alpha < \alpha_*$, since the integral curves joining the origin and the point $\alpha > \alpha_*$ exist in this case also. Compression of the continuous target can also occur at values $\alpha > \alpha_*$, since the integral curves joining the origin to the point ($V = \varkappa/3$, $z = \infty$) also exist in this case. The limiting value $\alpha = \alpha_{**}$ is defined by the condition of existence of singular points on the parabola $z = (V - \delta)^2$

$$\alpha_{**} = \frac{2}{\gamma - 1}\frac{2}{(1 + \sqrt{2/(\gamma-1)})^2}.$$

At ${}^5/_3 < \gamma < \gamma_1$, where $\gamma_1 = 2.412$ [7], on the other hand, for a hollow target α_{**} is achievable, while for compression of a solid target it is necessary that $\alpha < \alpha_*$, since in the opposite case the point B will lie on the separatrix of the infinitely removed singular point ($V = \varkappa/3$, $z = \infty$). Thus, when γ is varied over the range of greatest interest the maximum value of α attainable by one or the other method is the value α_{**}, while at $\gamma = {}^5/_3$ $\alpha_* = \alpha_{**}$.

We will now demonstrate that to achieve the maximum value of $\langle\rho R\rangle$ at time $t = 0$ for specified values of applied energy and compressible material mass it is necessary to realize a flow described by a self-similar solution at the maximum possible α value.

Generally speaking, the function $\langle\rho R\rangle$ reaches its maximum value at some time $t_* > 0$, but t_* is close to zero in value, so that at $t > 0$ intensive expulsion of gas from the target begins, and since the density at the center reaches its maximum value at $t = 0$ while the functions describing the state of the gas have the simplest analytical expressions, we will consider the problem of the maximum in $\langle\rho R\rangle$ at precisely this time.

Neglecting the target kinetic energy at $t = 0$ in comparison to its internal energy, from expressions for the mass M, internal energy E, and entropy function $S = p/\rho^\gamma$ we obtain

$$\langle\rho R\rangle = (ES)^{2\beta} M^{(\gamma-3)\beta} (\gamma - 1)^{2\beta} (3/4\pi)^{1/3} J(\alpha), \quad J(\alpha) = (3 - \alpha)^{(\gamma-3)\beta} (3 - \gamma\alpha)^{2\beta}/(1 - \alpha), \quad \beta = 1/3(\gamma - 1). \tag{1}$$

It follows from Eq. (1) that at $\alpha < 1$, $\gamma < 3$ $J(\alpha)$ is a monotonically increasing function of α, i.e., the maximum value of $\langle\rho R\rangle$ is reached at the maximum value $\alpha = \alpha_{**}$, achievable for a given value of γ. At $\alpha \geq 1$ $\langle\rho R\rangle$ tends to infinity, but as easily can be seen, the admissible values of α_*, α_{**} are such that $\alpha_{**} \geq 1$ at $\gamma < 1.34$, $\alpha_* \geq 1$ at $\gamma < 1.31$, i.e., at large values of γ an infinite $\langle\rho R\rangle$ cannot be obtained at finite energy expenditure and compressible material mass, the maximum value of $J(\alpha)$ being, for example, at $\gamma = {}^5/_3$, equal to $J(\alpha_*) \approx 5$, which is 3-4 times greater than the quantity $J(0) \approx 1.4$, which corresponds to constant density. Thus, we note that the efficiency of compression can be improved significantly in comparison to compression with homogeneous mass distribution over the target if we ensure a density distribution with exponent α_{**} at the moment of maximum compression. This is generally possible if the initial conditions correspond to a self-similar solution. For simpler, constant, initial conditions we may expect an asymptotic approach to the self-similar solution in the collapsing shell regime if the law of piston motion corresponds to this solution. This conclusion is based on results of numerical study of the well known problem of collapse of a spherical cavity, in which the solution exits to a self-similar regime independent of the initial data, which can be varied within certain limits. However, it is also possible to obtain a density distribution with exponent α_* at $t = 0$ for compression of a solid target with constant initial conditions. A self-similar solution with self-similarity index $\delta = 1$ describing compression of material to a point from a homogeneous original state was considered in [1-3]. The asymptotic distribution of the gasdynamic functions which develops upon perturbation of this solution by halting the piston was obtained in [2]. We will demonstrate below that an entire class of flows exist in which perturbation by halting the compressing piston leads to the same asymptotic parameter distribution as obtained in [2]. To do this we will consider isentropic flows in which the characteristics going toward the center intersect at the point ($r = 0$, $t = 0$). Representing the family of intersecting characteristics in the vicinity of the origin in the form $r = ct^\delta$, i.e., assuming that the relationship $u = -a + \delta r/t$ is satisfied, we obtain together with two equations describing isentropic flows a redefined system for the two unknowns u and a, the particle velocity and the speed of sound. After simple but lengthy transformations which we will not present here we obtain the condition for superposability of this system

$$\delta = \frac{2}{3\gamma - 1} + \sqrt{3}\frac{\gamma - 1}{3\gamma - 1}, \quad u \approx \frac{2}{3\gamma - 1}\frac{r}{t}, \quad a \approx \sqrt{3}\frac{\gamma - 1}{3\gamma - 1}\frac{r}{t}, \tag{2}$$

i.e., all flows in the region where the assumption $u = -a + \delta r/t$ is valid are described asymptotically by one and the same solution: the self-similar solution corresponding to the singular point B. According to this solution at $t = 0$ all the material is compressed to a point, and the density along every characteristic increases infinitely with approach to the center. As was shown in [5], to compress material to a point at $p \neq 0$ it is necessary to expend an infinite amount of energy, i.e., at a finite energy level it is impossible to accomplish such solutions up to the point of focusing, and up until that time the piston must unavoidably change its law of motion, thus creating a rarefaction wave the front of which moves toward the center along one of the characteristics [2]. But since the density along the characteristic increases without limit, given the condition of continuity of the motion the density distribution along the radius will have a singularity at the center at $t = 0$. If we represent this distribution as a power function, then, as was indicated above, the flow in the vicinity of the center will be self-similar, and since this solution must merge with solution (2) along the characteristic, it thus defines the self-similarity index δ uniquely, it being necessary that this value coincide with δ as obtained from Eq. (2). Hence the asymptote of the gasdynamic functions is uniquely defined at $t = 0$

$$u \sim a \sim r^{\mu(\gamma-1)},\ \rho \sim r^{2\mu},\ p \sim r^{2\mu\gamma},\ \mu = (\sqrt{3} - 3)/(2 + \sqrt{3}(\gamma - 1)). \tag{3}$$

Thus, all compression modes based on solutions with characteristics converging toward the center have one and the same asymptote, Eq. (3), at $t = 0$, independent of the concrete form of the solution, with the exponent to which ρ is raised coinciding with the value calculated previously, α_*. Thus, to reach a density distribution with exponent α_* at $t = 0$, it is sufficient to move the piston until its halt by any law which ensures compression of the material into a point with characteristics converging at the center.

Of all flows of this type, which include, for example, flows described by certain exact self-similar solutions [1-3, 6] and flows with homogeneous deformation [4-6], we must single out the case of self-similar flow with index $\delta = 1$, studied in [1-3], since it corresponds to constant initial data p_0, ρ_0, $u_0 = 0$. We will note that there also exist exact self-similar solutions which are a combination of two different solutions which coincide along some characteristic. The first component solution, describing compression of the material to a point, will be the solution corresponding to the singular point B. To any characteristic of that solution we may join a solution described by an integral curve, which in the self-similar plane joins the node O ($V = 0$, $z = 0$) and the singular point B, which for a δ value satisfying condition (2) will lie on the parabola $z = (V - \delta)^2$. It is just such a composite solution which is described by the main term in the vicinity of the origin of any of the solutions considered above with a rarefaction wave traveling toward the origin along one of the characteristics.

In studying the problem of maximum $\langle\rho R\rangle$, it was assumed that in the final stage of compression the density is distributed by a power law so that the integral quantity $\langle\rho R\rangle$ is essentially determined by the immediate vicinity of the center, where the density increases infinitely, while the peripheral portion of the target produces a smaller contribution to the integral while perhaps encompassing a significant portion of the mass and energy. Thus, it is desirable to study the behavior of the function $\langle\rho R\rangle$ assuming that the central portion is described by a power function with one exponent, and the peripheral portion by another exponent different from the first, i.e., the density is described by functions $\rho = c_1/r^{\alpha_1}$, $\rho = c_2/r^{\alpha_2}$ which are equal to each other at some point r_1 along the target radius r_2. Then, using the same notation as above, the following expression may be written for $\langle\rho R\rangle$:

$$\langle\rho R\rangle = (ES)^{2\beta} M^{\beta(\gamma-3)}(\gamma - 1)^{2\beta}(3/4\pi)^{1/3} J(\alpha_1, \alpha_2),\ \eta = r_2/r_1,$$
$$J(\alpha_1,\alpha_2) = \left[\frac{1}{1-\alpha_1} + \frac{1}{1-\alpha_2}(\eta^{1-\alpha_2} - 1)\right]\left[\frac{1}{3-\alpha_1} + \frac{1}{3-\alpha_2}(\eta^{3-\alpha_2} - 1)\right]^{(3-\gamma)\beta}\left[\frac{1}{3-\gamma\alpha_1} + \frac{1}{3-\gamma\alpha_2}(\eta^{3-\gamma\alpha_2} - 1)\right]^{-2\beta} \tag{4}$$

(the expression for $J(\alpha_1, \alpha_2)$ is corrected in a similar manner for $\alpha_i = 1, 3/\gamma, 3$).

We note that the density distribution function considered can be obtained by admitting a weak discontinuity only on the characteristic traveling from the piston to the point r_1. To do this it is necessary that the derivative functions u and a on both sides of the discontinuity (which we will denote by superscripts + and -) be matched, for example $u_r^+ - u_r^- = 2(a_r^+ - a_r^-)/(\gamma - 1)$, and the corresponding relationships must be satisfied for all higher derivatives. The derivatives found to the right of the discontinuity may be used to construct a formal series corresponding to the solution at $t < 0$, i.e., the compression process, if no shock wave is formed before the piston meets the characteristic. Considering the approximate nature of the expression for $\rho(r)$, there is no point in precise study of the question of shock wave formation, if we assume it possible to smooth the function in the immediate vicinity of the point r_1 when necessary.

Analysis of Eq. (4) reveals that at $\alpha_2 > \alpha_1$ the efficiency of compression increases, i.e., $J(\alpha_1, \alpha_2) > J(\alpha_1)$, but compression of a solid target by the method based on the solution described above with self-similarity index $\delta = 1$ at $\alpha_2 > \alpha_1$ is impossible, since the constant c_1 must then exceed the value permitted by the energy specified. The latter can easily be demonstrated, since the solution in the vicinity of the origin is known and described by Eq. (2), and thus improvement of the technique described is impossible. In collapse of hollow target the constant c_1 can be increased for the same total applied energy since the energy can be expended almost totally in piston work up to the moment when the piston trajectory intersects the singular characteristic going toward the center. After the piston is halted, the rarefaction wave produces a density distribution in the peripheral portion of the target with $\alpha_2 > \alpha_1$, so that the efficiency of hollow target compression is increased.

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MATHEMATICAL MODELING AND CALCULATION OF EXPLOSION EFFECT IN CONTINUOUS MEDIA

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In the conversion of explosion energy into electrical or mechanical energy, the following problem arises. There exists a plasma sphere of radius r_*, characterized by parameters p_0, ρ_0, T_0. At time $t = 0$ there occurs an instantaneous expulsion of photons and high velocity microparticles from this sphere, and the sphere also begins to expand into a spherical cavity, the space outside which is occupied by a continuous medium with parameters p_1, ρ_1, T_1 (Fig. 1). The surrounding medium is assumed condensed and will be studied using a hydrodynamic description. We will also assume that the surrounding medium effectively absorbs the energy of particles formed during the explosion, so that the major portion of the explosion energy is transferred to the medium in some region about the center of energy liberation. To define the parameters of the motion which develops it is necessary to develop a mathematical model of the flow to be studied, i.e., to write equations of motion for the continuous medium interacting with the particles and light radiation, and to specify initial and boundary conditions.

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